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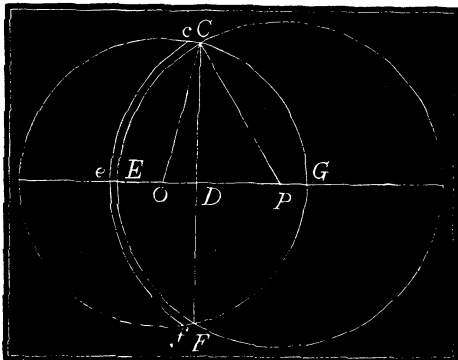
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# TO FIND THE AREA COMMON TO TWO INTERSECTING CIRCLES.

BY ARTEMAS MARTIN, MATHEMATICAL EDITOR SCHOOLDAY MAGAZINE.

Let  $O$  and  $P$  be the centers of two intersecting circles. Put  $OP = a$ ,  $OC = r$ , and suppose the radius  $CP$  variable and  $= x$ . The circles will intersect if  $x$  is not greater than  $r + a$  nor less than  $r - a$ . With center  $P$  and radius  $x + dx$  describe the arc  $ce f$  indefinitely near  $CE F$ . Then



$$DP = \frac{x^2 + a^2 - r^2}{2a}, \text{ arc } CEF = 2x \cos^{-1} \left( \frac{x^2 + a^2 - r^2}{2ax} \right)$$

and the differential of the area  $CEFG$  is

$$2x \cos^{-1} \left( \frac{x^2 + a^2 - r^2}{2ax} \right) dx.$$

Putting  $\Delta$  for the area sought, we have

$$\Delta = \int_{r-a}^x 2x \cos^{-1} \left( \frac{x^2 + a^2 - r^2}{2ax} \right) dx.$$

$$\int 2x \cos^{-1} \left( \frac{x^2 + a^2 - r^2}{2ax} \right) dx = x^2 \cos^{-1} \left( \frac{x^2 + a^2 - r^2}{2ax} \right)$$

$$+ \int \frac{x(r^2 - a^2 + x^2)dx}{\sqrt{4a^2r^2 - (r^2 + a^2 - x^2)^2}}.$$

Put  $r^2 + a^2 - x^2 = y$ , then

$$\int \frac{x(r^2 - a^2 + x^2)dx}{\sqrt{4a^2r^2 - (r^2 + a^2 - x^2)^2}} = \int \frac{-\frac{1}{2}(2r^2 - y)dy}{\sqrt{4a^2r^2 - y^2}}$$

$$= \int \frac{-r^2 dy}{\sqrt{4a^2r^2 - y^2}} + \int \frac{\frac{1}{2}y dy}{\sqrt{4a^2r^2 - y^2}} = r^2 \cos^{-1} \left( \frac{y}{4ar} \right) - \frac{1}{2} \sqrt{4a^2r^2 - y^2}.$$

$$\therefore \Delta = x^2 \cos^{-1} \left( \frac{x^2 + a^2 - r^2}{2 a x} \right) + r^2 \cos^{-1} \left( \frac{a^2 + r^2 - x^2}{2 a r} \right) - a \sqrt{r^2 - \left( \frac{a^2 + r^2 - x^2}{2 a} \right)^2}.$$

When  $x = R$ ,

$$\Delta = R^2 \cos^{-1} \left( \frac{a^2 - r^2 + R^2}{2 a R} \right) + r^2 \cos^{-1} \left( \frac{a^2 + r^2 - R^2}{2 a r} \right) - a \sqrt{r^2 - \left( \frac{a^2 + r^2 - R^2}{2 a} \right)^2},$$

which agrees with the result obtained by the ordinary method.

The above formula may be readily adapted to any special case.

When the center  $P$  is on the circumference of the other circle,  $a = r$ , and

$$\Delta = R^2 \cos^{-1} \left( \frac{R}{2 r} \right) + 2 r^2 \sin^{-1} \left( \frac{R}{2 r} \right) - \frac{1}{2} R \sqrt{4 r^2 - R^2}.$$

If the circles are equal,  $R = r$  and

$$\Delta = 2 r^2 \cos^{-1} \left( \frac{a}{2 r} \right) - \frac{1}{2} a \sqrt{4 r^2 - a^2}.$$

When they are equal and the center of one on the circumference of the other,  $R = r$ ,  $a = r$  and

$$\Delta = r^2 \left( \frac{2}{3} \pi - \frac{1}{2} \sqrt{3} \right).$$

### PROBLEMS.

5. In a plane triangle there are given the three lines bisecting the angles,  $a$ ,  $b$  and  $c$ , to find the sides.—Communicated by DR. DAVID S. HART, Stonington, Conn.

6. Find a convenient formula for calculating the capacity of a cistern constructed as follows, viz: Having a lower concavity which is a spherical segment whose versed sine is  $a$  and chord  $2 r$ , a central cylindrical part whose radius is  $r$  and perpendicular height  $h$ , and an upper concavity which is a spherical segment whose versed sine is  $b$  and chord  $2 r$ .—Communicated by FRANK PELTON, C. E., Des Moines, Iowa.

$$7. \text{ Multiply } \sqrt[3]{1 + \sqrt[3]{1 + \sqrt[3]{1 + \&c.}}} \text{ by } \frac{\sqrt{1 + 1}}{\sqrt{1 + \&c.}}$$

and express the product in a finite number of terms.—Communicated by PROF. DANIEL KIRKWOOD, Bloomington, Ind.